Difficulty of the Conventional Approach

- Lack of invariance to the baseline condition
- Inference depends on the choice of baseline condition
- 3×2 example:
  - Treatment $A \in \{a_0, a_1, a_2\}$ and Treatment $B \in \{b_0, b_1, b_2\}$
  - Regression model with the baseline condition (a_0, b_0):

$$\mathbb{E}(Y \mid A, B) \quad = \quad 1 + a_1^* + a_2^* + b_2^* + a_1^* b_2^* + 2a_2^* b_2^* + 3a_2^* b_1^*$$

  - Interaction effect for $(a_2, b_2) \; > \;$ Interaction effect for (a_1, b_2)
  - Another equivalent model with the baseline condition (a_0, b_1):

$$\mathbb{E}(Y \mid A, B) \quad = \quad 1 + a_1^* + 4a_2^* + b_2^* + a_1^* b_2^* - a_2^* b_2^* - 3a_2^* b_0^*$$

  - Interaction effect for $(a_2, b_2) \; < \;$ Interaction effect for (a_1, b_2)
  - Interaction effect for (a_2, b_1) is zero under the second model
  - All interaction effects with at least one baseline value are zero

The Contributions of the Paper

1. Standard treatment interaction effects suffer from the lack of order and interval invariance to the choice of baseline condition
2. Propose the marginal treatment interaction effect that is invariant
3. Derive the identification condition and estimation strategy for this new quantity
4. Generalize these results to the K-way causal interaction
5. Illustrate the methods with the immigration survey experiment

Two-way Causal Interaction

- Two factorial treatments:

$$\begin{aligned} A &\in \mathcal{A} = \{a_0, a_1, \ldots, a_{D_A-1}\} \\ B &\in \mathcal{B} = \{b_0, b_1, \ldots, b_{D_B-1}\} \end{aligned}$$

- Assumption: Full factorial design
  1. Randomization of treatment assignment

$$\{Y(a_\ell, b_m)\}_{a_\ell \in \mathcal{A}, b_m \in \mathcal{B}} \quad \perp\!\!\!\perp \quad \{A, B\}$$

  2. Non-zero probability for all treatment combination

$$\Pr(A = a_\ell, B = b_m) \quad > \quad 0 \quad \text{for all } a_\ell \in \mathcal{A} \quad \text{and} \quad b_m \in \mathcal{B}$$

- Fractional factorial design not allowed
  1. Use a small non-zero assignment probability
  2. Focus on a subsample
  3. Combine treatments